Problem 1
Transforming \( y = x^3 \)

What is an equation of the graph of \( y = x^3 \) under a vertical compression by the factor \( \frac{1}{2} \) followed by a reflection across the \( x \)-axis, a horizontal translation 3 units to the right, and then a vertical translation 2 units up?

**Step 1** Multiply by \( \frac{1}{2} \) to compress.

\[
y = x^3 \quad \rightarrow \quad y = \frac{1}{2}x^3
\]

**Step 2** Multiply by \( -1 \) to reflect.

\[
y = \frac{1}{2}x^3 \quad \rightarrow \quad y = -\frac{1}{2}x^3
\]

**Step 3** Replace \( x \) with \( x - 3 \) to translate horizontally.

\[
y = -\frac{1}{2}x^3 \quad \rightarrow \quad y = -\frac{1}{2}(x - 3)^3
\]

**Step 4** Add 2 to translate vertically.

\[
y = -\frac{1}{2}(x - 3)^3 \quad \rightarrow \quad y = -\frac{1}{2}(x - 3)^3 + 2
\]
**Got It? 1.** What is an equation of the graph of \( y = x^3 \) under a vertical stretch by the factor 2 followed by a horizontal translation 3 units to the left and then a vertical translation 4 units down?

The graph shows \( y = x^3 \) and the graphs that result from the transformations in Problem 1.

In general, \( y = a(x - h)^3 + k \) represents all of the cubic functions you can obtain by stretching, compressing, reflecting, or translating the cubic parent function \( y = x^3 \).

**Problem 2** Finding Zeros of a Transformed Cubic Function

**Multiple Choice** If \( a, h, \) and \( k \) are real numbers and \( a \neq 0 \), how many distinct real zeros does \( y = -a(x - h)^3 + k \) have?

- \( A \) 0
- \( B \) 1
- \( C \) 2
- \( D \) 3

\[-a(x - h)^3 + k = 0 \]
\[-a(x - h)^3 = -k \] Subtract \( k \) from each side.
\[(x - h)^3 = \frac{k}{a} \] Divide each side by \( -a \).
\[x - h = \sqrt[3]{\frac{k}{a}} \] Take the cube root of each side.
\[x = \sqrt[3]{\frac{k}{a}} + h \] Solve for \( x \).

Disregarding multiplicities, the function has a single real zero. The correct answer is B.

**Got It? 2.** What are all the real zeros of the function \( y = 3(x - 1)^3 + 6 \)?

Problems 1 and 2 together illustrate that the graph of an “offspring” function of the parent cubic function \( y = x^3 \) has only one \( x \)-intercept.

The graph of the cubic function \( y = x^3 - 2x^2 - 5x + 6 \) has three \( x \)-intercepts. You cannot obtain this function or others like it by transforming the parent cubic function \( y = x^3 \) using stretches, reflections, and translations.

Similarly, some quartic functions are simple transformations of \( y = x^4 \) and some are not.
What is a quartic function with only two real zeros, \( x = 5 \) and \( x = 9 \)?

**Method 1** Use transformations.

First, find a quartic with zeros at \( \pm 2 \).

Translate the basic quartic 16 units down:

\[
y = x^4 \rightarrow y = x^4 - 16
\]

9 is 7 units to the right of 2.

Translate 7 units to the right.

\[
y = x^4 - 16 \rightarrow y = (x - 7)^4 - 16
\]

A quartic function with its only real zeros at 5 and 9 is 
\[ y = (x - 7)^4 - 16. \]

**Method 2** Use algebraic methods.

\[
y = (x - 5)(x - 9) \cdot Q(x)
\]

\[
= (x - 5)(x - 9)(x^2 + 1)
\]

\[
= (x^2 - 14x + 45)(x^2 + 1)
\]

\[
= x^4 - 14x^3 + 46x^2 - 14x + 45
\]

Another quartic function with its only real zeros at 5 and 9 is 
\[ y = x^4 - 14x^3 + 46x^2 - 14x + 45. \]

The “offspring” of the parent function \( y = x^4 \) is a subfamily of all quartic polynomials. This subfamily consists of quartics of the form \( y = a(x - h)^4 + k \). These functions also belong to another category of polynomials, and in this category you can generate families as usual.

**Definition**

A **power function** is a function of the form

\[ y = a \cdot x^b, \]

where \( a \) and \( b \) are nonzero real numbers.

**Examples**

\[
y = 0.5x^6
\]

\[
y = \frac{1}{2}x^2
\]

\[
y = -4x^{\frac{1}{2}}
\]

\[
y = x^{0.25}
\]

If the exponent \( b \) in \( y = ax^b \) is a positive integer, the function is also a **monomial function**.

If \( y = ax^b \) describes \( y \) as a power function of \( x \), then \( y \) varies directly with, or is proportional to, the \( b \)th power of \( x \). The constant \( a \) is the **constant of proportionality**.

Power functions arise in many real-world contexts related to the concept of direct variation, which you studied in Chapter 2.
Problem 4  Modeling With a Power Function

Wind-Generated Power  Wind farms are a source of renewable energy found around the world. The power $P$ (in kilowatts) generated by a wind turbine varies directly as the cube of the wind speed $v$ (in meters per second). The picture shows the power output of one turbine at one wind speed. To the nearest kilowatt, how much power does this turbine generate in a 10 m/s wind?

The formula for $P$ as a power function of $v$ is $P = a \cdot v^3$. From the picture, $P = 600$ when $v = 8$, or $600 = a \cdot 8^3$. Solve for $a$.

$$600 = a \cdot 8^3$$  Use values of $P$ and $v$ to find $a$.

$$600 = 512a$$

$$a = 1.1719$$

$$P = 1.1719v^3.$$  Use the value of $a$ in the original formula.

$$P = 1.1719 \cdot 10^3 = 1171.9.$$  Substitute 10 for $v$ and simplify.

This turbine generates about 1172 kW of power in a 10 m/s wind.

Got It?  4. Another turbine generates 210 kW of power in a 12 mi/h wind. How much power does this turbine generate in a 20 mi/h wind?

Lesson Check

Do you know HOW?

Find all the real zeros of each function.

1. $y = -(x + 3)^3 + 1$
2. $y = -8(x - 5)^3 - 64$
3. $y = \frac{9}{2}(x - 1)^3 + \frac{4}{3}$

Do you UNDERSTAND?

4. Vocabulary  Is the function $y = 4x^3 + 5$ an example of a power function? Explain.

5. Error Analysis  Your friend says that he has found a way to transform the graph of $y = x^3$ to obtain three real roots. Using the graph of the function, explain why this is impossible.

6. Compare and Contrast  How are the graphs of $y = x^3$ and $y = 4x^3$ alike? How are they different? What transformation was used to get the second equation?
Determine the cubic function that is obtained from the parent function \( y = x^3 \) after each sequence of transformations.

7. a vertical stretch by a factor of 3; a reflection across the \( x \)-axis; a vertical translation 2 units up; and a horizontal translation 1 unit right

8. a vertical stretch by a factor of 2; a vertical translation 4 units up; and a horizontal translation 3 units left

9. a reflection across the \( y \)-axis; a vertical translation 1 unit down; and a horizontal translation 5 units left

10. a vertical translation 3 units down; and a horizontal translation 2 units right

11. a vertical stretch by a factor of 3; a reflection across the \( y \)-axis; a vertical translation \( \frac{3}{2} \) unit up; and a horizontal translation \( \frac{1}{2} \) unit left

12. a vertical stretch by a factor of \( \frac{5}{3} \); a reflection across the \( x \)-axis; a vertical translation 4 units down; and a horizontal translation 3 units right

Find all the real zeros of each function.

13. \( y = -27(x - 2)^3 + 8 \)

14. \( y = -\frac{1}{8}(x - 7)^3 - 8 \)

15. \( y = -3(x + \frac{4}{3})^3 + \frac{8}{9} \)

16. \( y = -16(x + 3)^3 + 9 \)

17. \( y = 4(x - 1)^3 + 10 \)

18. \( y = 2(x + 5)^3 + 10 \)

Find a quartic function with the given \( x \)-values as its only real zeros.

19. \( x = 2 \) and \( x = -1 \)

20. \( x = -3 \) and \( x = -4 \)

21. \( x = -1 \) and \( x = 3 \)

22. \( x = 4 \) and \( x = 2 \)

23. \( x = -4 \) and \( x = -1 \)

24. \( x = -3 \) and \( x = 2 \)

25. **Cooking** The number of pepperoni slices that Kim puts on a pizza varies directly as the square of the diameter of the pizza. If she puts 15 slices on a 10" diameter pizza, how many slices should she put on a 16" diameter pizza?

26. **Volume** The amount of water that a spherical tank can hold varies directly as the cube of its radius. If a tank with radius 7.5 ft holds 1767 ft\(^3\) of water, how much water can a tank with radius 16 ft hold?

27. **Think About a Plan** The kinetic energy generated by a 5 lb ball is represented by the formula \( K = \frac{1}{2}(5)v^2 \). If the ball is thrown with a velocity of 6 ft/sec, how much kinetic energy is generated?

- What does 5 represent in the function?
- What number should you substitute for \( v \)?
Determine whether each function can be obtained from the parent function, \( y = x^n \), using basic transformations. If so, describe the sequence of transformations.

28. \( y = -4x^3 \)

29. \( y = 2(x - 3)^2 + 5 \)

30. \( y = x^3 - x \)

31. \( y = x^2 - 8x + 7 \)

Determine the transformations that were used to change the graph of the parent function \( y = x^3 \) to each of the following graphs.

32. 33. 34.

35. 36. 37.

38. Compare the function \( y = 3x^3 \) to the function shown in the graph at the right. Which function has a greater vertical stretch factor? Explain.

39. **Physics** The formula \( K = \frac{1}{2}mv^2 \) represents the kinetic energy of an object. If the kinetic energy of a ball is 10 lb*ft^2/s^2 when it is thrown with a velocity of 4 ft/s, how much kinetic energy is generated if the ball is thrown with a velocity of 8 ft/s?

40. **Reasoning** Explain why the basic transformations of the parent function \( y = x^3 \) will only generate functions that can be written in the form \( y = a(x - h)^3 + k \).

41. **Reasoning** Explain why some quartic polynomials cannot be written in the form \( y = a(x - h)^4 + k \). Give two examples.
Lesson 5-9
Transforming Polynomial Functions

42. **Reasoning** Find a sequence of basic transformations by which the polynomial function \( y = 2x^3 - 6x^2 + 6x + 5 \) can be derived from the cubic function \( y = x^3 \).

43. **Physics** For a constant resistance \( R \) (in ohms), the power \( P \) (in watts) dissipated across two terminals of a battery varies directly as the square of the current \( I \) (in amps). If a battery connected in a circuit dissipates 24 watts of power for 2 amps of current flow, how much power would be dissipated when the current flow is 5 amps?

44. **Writing** Give an argument that shows that every polynomial family of degree \( n > 2 \) contains polynomials that cannot be generated from the basic function \( y = x^n \) by using stretches, compressions, reflections, and translations.

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**Standardized Test Prep**

Use the graph to answer questions 45–47.

45. Which equation does the graph represent?
   - A) \( y = (x + 2)^2 - 1 \)
   - B) \( y = (x - 2)^2 - 1 \)
   - C) \( y = (x - 2)^2 + 1 \)
   - D) \( y = (x - 2)^4 - 1 \)

46. If \( y = f(x) \) is an equation for the graph, what are factors of \( f(x) \)?
   - F) \( (x - 1) \) and \( (x + 3) \)
   - G) \( (x - 1) \) and \( (x - 3) \)
   - H) \( (x + 1) \) and \( (x - 3) \)
   - I) \( (x + 1) \) and \( (x + 3) \)

47. If \( y = ax^2 + bx + c \) is an equation for the graph, what type of number is its discriminant?

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**Mixed Review**

Find a polynomial function whose graph passes through the given points.

48. \((-1, 4), (0, -2), (1, -2), (2, -8)\)
49. \((-2, -17), (0, -3), (1, -5), (3, 63)\)

Write an equation of each line.

50. slope = \(-\frac{4}{3}\); through \((-1, 4)\)
51. slope = \(-3\); through \((2, -1)\)

Determine whether each relation is a function.

52. \{(-1, -1), (-3, 2), (2, 3), (-3, 3)\}
53. \{(-4, 0), (-7, 0), (-4, 1), (-7, 1)\}

**Get Ready!** To prepare for Lesson 6-1, do Exercises 54–56.

Factor each expression.

54. \(x^{10} + x^2\)
55. \(x^4 - y^4\)
56. \(169x^6y^{12} - 13x^3y^6\)

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